Ratios of B and D Meson Decay Constants in Improved Mock Meson Model

Dae Sung Hwang[1] and Gwang-Hee Kim[2]

Department of Physics, Sejong University, Seoul 143–747, Korea

Abstract

We calculate the ratio f_{B_s}/f_{B_d} by following the Oakes' method which is based on chiral symmetry breaking, but we improve his calculation by performing the calculation of the factor $< 0|\bar{b}\gamma_5 s|B_s >/< 0|\bar{b}\gamma_5 d|B_d >$ in the mock meson model. In this calculation we improved the mock meson model by using the value of the parameter β which is obtained by the variational method in the relativistic quark model. We also calculate f_{D_s}/f_{D_d}, and then $(f_{B_s}/f_{B_d})\,/\,(f_{D_s}/f_{D_d})$. In this method we also obtain the ratio f_{B_s}/f_{D_s} and f_{B_d}/f_{D_d} which are important for the knowledge of CP violation and B-$\bar{B}$ mixing.

1: e-mail: dshwang@phy.sejong.ac.kr

2: e-mail: gkim@phy.sejong.ac.kr

Oakes [1] calculated the ratio f_{B_s}/f_{B_d} based on the usual assumption that chiral symmetry is broken by quark mass terms $H_1(x) = \sum m_i \bar{q}_i(x) q_i(x)$. Using the local relation $\partial^\mu A_\mu(x) = -i[Q_5, H_1(x)]$, he obtained

$$\frac{f_{B_s}}{f_{B_d}} = \Big(\frac{M_{B_d}}{M_{B_s}}\Big)^2 \Big(\frac{m_b + m_s}{m_b + m_d}\Big) \frac{< 0|\bar{b}\gamma_5 s|B_s >}{< 0|\bar{b}\gamma_5 d|B_d >}, \tag{1}$$

where the decay constants are defined by

$$< 0|\bar{b}\gamma^\mu\gamma_5 s|B_s(\mathbf{K}) >= i f_{B_s} K^\mu, \quad < 0|\bar{b}\gamma^\mu\gamma_5 d|B_d(\mathbf{K}) >= i f_{B_d} K^\mu. \tag{2}$$

Through the symmetry consideration, he took the value of $< 0|\bar{b}\gamma_5 s|B_s >/< 0|\bar{b}\gamma_5 d|B_d >$ as 1, and then obtained the results

$$f_{B_s}/f_{B_d} = 0.989, \quad f_{D_s}/f_{D_d} = 0.985, \quad (f_{B_s}/f_{B_d})\,/\,(f_{D_s}/f_{D_d}) = 1.004. \tag{3}$$

Because of the light quark SU(3) flavor symmetry the factor $< 0|\bar{b}\gamma_5 s|B_s >/< 0|\bar{b}\gamma_5 d|B_d >$ is almost 1. However, since we are interested in the ratios in (3) which are by themselves very close to 1, the deviation of the above factor from 1 is still important even though it is very small. Therefore it would be nice if we could calculate the small correction of the symmetry consideration. In this Letter we calculate this factor in the mock meson model of Capstick, Godfrey, and Isgur [2, 3, 4], and improve the Oakes' results in (3). We also apply this approach to the calculations of f_{B_s}/f_{D_s} and f_{B_d}/f_{D_d}, where the calculations of the above factor are essential since we can not apply the light quark flavor symmetry in these cases. The knowledge of these ratios are important since they allow us to get the values of f_{B_s} and f_{B_d} from the experimentally obtained value of f_{D_s}. The information of f_{B_s} and f_{B_d} is very important, since it is crucial for the magnitude of CP violation and the size of B-$\bar{B}$ mixing.

The mock meson state is represented by [2, 3, 4]

$$|M(\mathbf{K}) >= \int d^3p\, \Phi(\mathbf{p})\, \chi_{s\bar{s}}\, \phi_{c\bar{c}}\, |q(\frac{m_q}{m}\mathbf{K} + \mathbf{p}, s)\, \bar{Q}(\frac{m_{\bar{Q}}}{m}\mathbf{K} - \mathbf{p}, \bar{s}) >, \tag{4}$$

where $\mathbf{K}$ is the mock meson momentum, $m \equiv m_q + m_{\bar{Q}}$, and $\Phi(\mathbf{p})$, $\chi_{s\bar{s}}$, and $\phi_{c\bar{c}}$ are momentum, spin, and color wave functions respectively. We take the momentum wave function $\Phi(\mathbf{p})$ as a Gaussian wave function

$$\Phi(\mathbf{p}) = \frac{1}{(\sqrt{\pi}\beta)^{3/2}} e^{-\mathbf{p}^2/2\beta^2}. \tag{5}$$

We note that we normalized the mock meson state $|M(\mathbf{K}) >$ according to $< M(\mathbf{K}')|M(\mathbf{K}) >= \delta^3(\mathbf{K}' - \mathbf{K})$ [5], and we consider only pseudoscalar mesons in the present work. In this normalization of the mock meson state, (2) becomes [5]

$$< 0|\bar{b}\gamma^\mu\gamma_5 s|B_s(\mathbf{K}) >= \frac{i f_{B_s} K^\mu}{\sqrt{2E_{B_s}}}, \quad < 0|\bar{b}\gamma^\mu\gamma_5 d|B_d(\mathbf{K}) >= \frac{i f_{B_d} K^\mu}{\sqrt{2E_{B_d}}}. \tag{6}$$

By applying the local relation $\partial^\mu A_\mu(x) = -i[Q_5, H_1(x)]$ to (6), we get

$$f_{B_s} = \frac{\sqrt{2E_{B_s}}(m_b + m_s)}{(M_{B_s})^2} < 0|\bar{b}\gamma_5 s|B_s(\mathbf{K}) >, \tag{7}$$

and a similar expression for f_{B_d}.

Then in the rest frames of the mesons we have

$$\frac{f_{B_s}}{f_{B_d}} = \Big(\frac{M_{B_d}}{M_{B_s}}\Big)^{3/2} \Big(\frac{m_b + m_s}{m_b + m_d}\Big) \frac{< 0|\bar{b}\gamma_5 s|B_s(\mathbf{0}) >}{< 0|\bar{b}\gamma_5 d|B_d(\mathbf{0}) >}, \tag{8}$$

where

$$\begin{aligned} < 0|\bar{Q}\gamma_5 q|P_{q\bar{Q}}(\mathbf{0}) >\equiv g(\beta) &= 2\sqrt{3}\int d^3p\, \Phi(\mathbf{p}) \Big(\frac{E_q + m_q}{2E_q}\,\frac{E_{\bar{Q}} + m_{\bar{Q}}}{2E_{\bar{Q}}}\Big)^{1/2} \\ &\quad \times \Big(1 + \frac{\mathbf{p}^2}{(E_q + m_q)(E_{\bar{Q}} + m_{\bar{Q}})}\Big). \end{aligned} \tag{9}$$

We note that the power of M_{B_d}/M_{B_s} in (8) is 3/2 instead of 2 in (1), since we use the normalization $< M(\mathbf{K}')|M(\mathbf{K}) >= \delta^3(\mathbf{K}' - \mathbf{K})$, whereas Oakes used the more common normalization $< M(\mathbf{K}')|M(\mathbf{K}) >= 2E\,\delta^3(\mathbf{K}' - \mathbf{K})$. We also note that the sign inside the last parenthesis in (9) is plus, whereas that in Eq. (3) of Ref. [3] is minus. This difference originates from the fact that we consider the pseudoscalar quantity of the left hand side of (9), whereas Ref. [3] considered the zeroth component of the four vector $< 0|\bar{q}\gamma^\mu(1 - \gamma_5)|M(\mathbf{K}) >$. In order to

calculate the ratio of the decay constants from (8), we should calculate the matrix element $< 0|\bar{Q}\gamma_5 q|P_{q\bar{Q}}(\mathbf{0})>$. This matrix element is a function of β, which we call $g(\beta)$. We performed numerical calculations for the $g(\beta)$ of B_s, B_d, D_s, and D_d respectively, and present the results in Fig. 1.

Then the problem is what values we should use for β. Capstick and Godfrey used the values of β obtained from the effective harmonic oscillator potential [3, 4]. Here, we improve this treatment by applying the variational method to the relativistic hamiltonian [6, 7]

$$H = \sqrt{\mathbf{p}^2 + {m_Q}^2} + \sqrt{\mathbf{p}^2 + {m_q}^2} + V(r), \tag{10}$$

where $\mathbf{r}$ and $\mathbf{p}$ are the relative coordinate and its conjugate momentum. The hamiltonian in (10) represents the energy of the meson in the meson rest frame, since in this reference frame the momentum of each quark is the same in magnitude as that of the conjugate momentum of the relative coordinate. In Ref. [7] we obtained the values of β which minimize the expectation value of the hamiltonian in (10). We considered six different potentials for $V(r)$ in (10), and the averages of the minimizing β values, $\bar{\beta}$, obtained from six potentials are given by [7]

$$\bar{\beta}_{B_s} = 0.579 \text{ GeV}, \quad \bar{\beta}_{B_d} = 0.558 \text{ GeV}, \quad \bar{\beta}_{D_s} = 0.492 \text{ GeV}, \quad \bar{\beta}_{D_d} = 0.475 \text{ GeV}. \tag{11}$$

In the above calculations we used the current quark masses of the light quarks as m_d=9.9 MeV and m_s=199 MeV, which were given by Dominguez and Rafael [8].

By using the values in (11) for $\bar{\beta}$, we can obtain from the $g(\beta)$ in Fig. 1 the values of the matrix elements $g(\bar{\beta}) \equiv < 0|\bar{Q}\gamma_5 q|P_{q\bar{Q}}(\mathbf{0})>$ in (9), which are given by

$$g_{B_s}(0.579) = 8.657, \quad g_{B_d}(0.558) = 7.494, \quad g_{D_s}(0.492) = 7.418, \quad g_{D_d}(0.475) = 6.533. \tag{12}$$

Then, using the meson [9] and the current quark [8] masses given by

$$M_{B_s} = 5.375\,\text{GeV},\ M_{B_d} = 5.279\,\text{GeV},\ M_{D_s} = 1.969\,\text{GeV},\ M_{D_d} = 1.869\,\text{GeV},$$
$$m_b = 4.397\,\text{GeV},\ m_c = 1.306\,\text{GeV},\ m_s = 0.199\,\text{GeV},\ m_d = 0.0099\,\text{GeV}, \tag{13}$$

we obtain the following ratios of the decay constants from (8).

$$\frac{f_{B_s}}{f_{B_d}} = 1.173, \quad \frac{f_{D_s}}{f_{D_d}} = 1.201, \quad \frac{f_{D_s}}{f_{B_s}} = 1.266, \quad \frac{f_{D_d}}{f_{B_d}} = 1.236. \tag{14}$$

When we use the values of the ratios in (14) together with the experimental value of f_{D_s} given by $f_{D_s} = 265 \pm 68$ MeV [7, 9, 10, 11], we obtain $f_{D_d} = 221 \pm 57$ MeV, $f_{B_s} = 209 \pm 54$ MeV, and $f_{B_d} = 178 \pm 46$ MeV.

In order to compare our results in (14) with the Oakes' original approach, we calculated the ratios using the formula (1) with our values of the meson and current quark masses in (13). Here, we took the last factor in (1) as 1, as Oakes did. The results are presented in the row 2 of Table 1. As we see in Table 1, our results of the ratios f_{B_s}/f_{B_d} and f_{D_s}/f_{D_d} have been enhanced significantly compared with the Oakes' results, since we have taken care of the meson structure through (8) and (9). However, the double ratio $(f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d})$ remains the same.

For comparisons, we summarize the results of various different calculations in Table 1. Grinstein did the calculation using the heavy quark effective theory, and Dominguez using the QCD sum rules. The row 6 is the results of the original calculations of Capstick and Godfrey. They used the values of the parameter β given by the effective harmonic oscillator potential, whereas we used the values given by the variational method in the relativistic quark model. Another important difference between their and our calculations is that they used the formula (2) directly and had the ambiguity of which component of the four vector should be used to obtain f_P as they discussed [4], whereas we followed the Oakes' approach based on chiral symmetry breaking and then arrived at the formula (8) which has no such ambiguity. In the row 7 we present the results which the authors of this Letter obtained in the relativistic quark model [7]. The rows 8-11 are the results of the lattice calculations, where the third column was estimated from their announced values of the first and second columns, and the fourth and fifth columns were estimated from their results of f_{D_s}, f_{B_s}, f_{D_d}, and f_{B_d}.

Acknowledgements

This work was supported in part by the Basic Science Research Institute Program, Ministry of Education, Project No. BSRI-94-2414, and in part by Daeyang Foundation at Sejong University in 1994.

References

[1] R.J. Oakes, Phys. Rev. Lett. 73 (1994) 381.

[2] S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189.

[3] S. Godfrey, Phys. Rev. D33 (1986) 1391.

[4] S. Capstick and S. Godfrey, Phys. Rev. D41 (1990) 2856.

[5] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989) 113.

[6] D.S. Hwang, C.S. Kim and W. Namgung, hep-ph-9502346, to appear in Z. Phys. C (1995).

[7] D.S. Hwang and G.-H. Kim, hep-ph-9507340 (1995).

[8] C.A. Dominguez and E. de Rafael, Ann. Phys. 174 (1987) 372; Y. Koide, hep-ph-9410270 (1994).

[9] Review of Particle properties, Phys. Rev. D50, 3-I (August 1994).

[10] S. Aoki *et al.*, WA75 collaboration, Prog. Theor. Phys. 89 (1993) 131.

[11] D. Acosta *et al.*, CLEO collaboration, Phys. Rev. D49 (1994) 5690.

[12] B. Grinstein, Phys. Rev. Lett. 71 (1993) 3067.

[13] C.A. Dominguez, in Proceedings of the Third Workshop on Tau-Charm Factory, Marbella, Spain, June, 1993.

[14] ELC: As. Abada *et al.*, Nucl. Phys. B376 (1992) 172.

[15] UKQCD: R.M. Boxter *et al.*, Phys. Rev. D49 (1994) 1594.

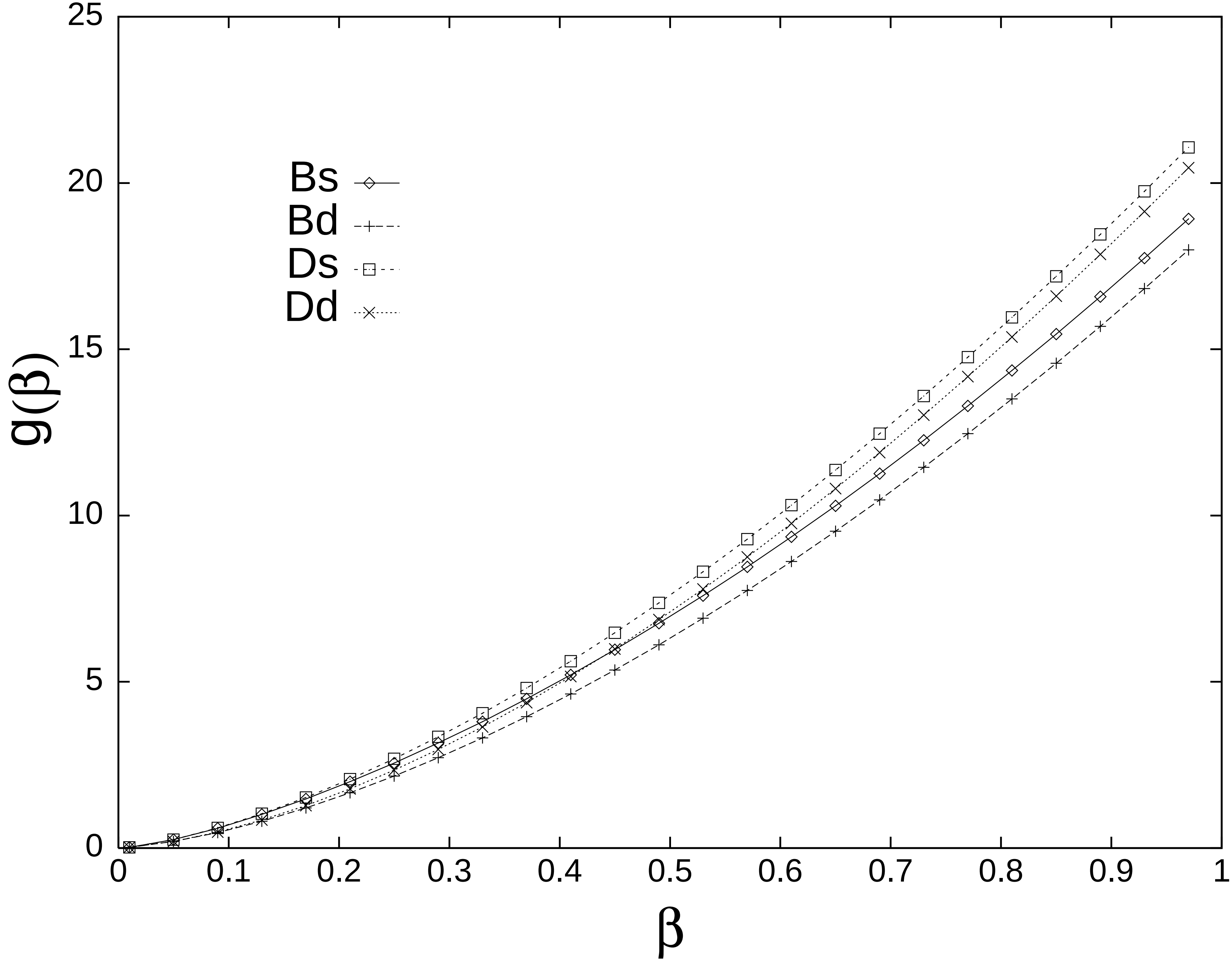
[16] BLS: C.W. Bernard, J.N. Labrenz and A. Soni, Phys. Rev. D49 (1994) 2536.

[17] MILC: C.W. Bernard *et al.*, FSU-SCRI-95C-28 (1995).

		f_{B_s}/f_{B_d}	f_{D_s}/f_{D_d}	$\frac{(f_{B_s}/f_{B_d})}{(f_{D_s}/f_{D_d})}$	f_{D_s}/f_{B_s}	f_{D_d}/f_{B_d}
1	This Work	1.173	1.201	0.977	1.266	1.236
2	Oakes Modif.	1.006	1.030	0.977	—	—
3	Oakes [1]	0.989	0.985	1.004	—	—
4	Grinstein [12]	—	—	0.967	—	—
5	Dominguez [13]	1.22±.02	1.21±.06	1.01±.05	—	—
6	Cap. Godf. [4]	1.35±.18	1.21±.13	1.12±.19	1.38±.16	1.55±.20
7	Hwang Kim [7]	1.053	1.045	1.008	1.251	1.261
8	ELC [14]	1.08±.06	1.08±.02	1.00±.06	1.03±.22	1.02±.21
9	UKQCD[15]	$1.22\,^{+.04}_{-.03}$	1.18±.02	$1.03\,^{+.04}_{-.03}$	$1.09\,^{+.04+.42}_{-.03-.06}$	$1.16\,^{+.05+.46}_{-.05-.14}$
10	BLS [16]	1.11±.02 ± .05	1.11±.02 ± .05	1.00±.03 ± .06	1.11±.06 ± .27	1.11±.08 ± .30
11	MILC [17]	1.13(2)(9)(4)	1.09(1)(4)(4)	1.04(2)(9)(5)	1.18(3)(17)(13)	1.22(5)(17)(19)

Table 1: The ratios of the decay constants from different calculations.

Fig. 1. $g_{B_s}(\beta) \equiv < 0|\bar{b}\gamma_5 s|B_s(\mathbf{0}) >$, $g_{B_d}(\beta) \equiv < 0|\bar{b}\gamma_5 d|B_d(\mathbf{0}) >$, $g_{D_s}(\beta) \equiv < 0|\bar{c}\gamma_5 s|D_s(\mathbf{0}) >$, and $g_{D_d}(\beta) \equiv < 0|\bar{c}\gamma_5 d|D_d(\mathbf{0}) >$, as functions of the parameter β.



25
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
g(β)
β
Bs
Bd
Ds
Dd